

F -term induced flavor mass spectrum

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ABSTRACT: New mechanism of generating flavor mass spectrum is proposed by using an O’Raifeartaigh-type supersymmetry breaking model. A desired bilinear form of fermion mass spectrum is naturally realized through F -components of gauge-singlet (nonet of SU(3) flavor symmetry) superfields, and the suitable charged-lepton mass relation is reproduced. The *charged-slepton* mass spectrum is non-degenerate in general, and can be even hierarchical (proportional to the charged-lepton masses in the specific case). Flavor changing neutral processes are suppressed since the charged-lepton and *slepton* (except for right-handed sneutrino) mass matrices are diagonalized simultaneously in the flavor space. The right-handed sneutrinos are light with the similar ratio to the lepton sector ($\tilde{m}_{\nu_R}/\tilde{m}_e \sim m_\nu/m_e$).

KEYWORDS: Supersymmetry Breaking, Quark Masses and SM Parameters, Neutrino Physics.

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1. Introduction

Investigating an origin of a flavor mass spectrum will provide an important clue to the underlying theory of quarks and leptons. In the standard model, the flavor mass spectrum is originated in the structure of Yukawa coupling constants Y_{ij}^f ($f = u, d, \nu, e; i, j = 1, 2, 3$), where fermion mass matrices M^f are given by $M_{ij}^f = Y_{ij}^f \langle H \rangle$ (H is the Higgs doublet). When there is a flavor symmetry in the mass matrices M^f , we have predictions for the masses and mixings.

On the other hand, there is another idea for the origin of the flavor mass spectrum, which is originated in a structure of vacuum expectation values (VEVs) of scalar fields (all couplings are of $\mathcal{O}(1)$). This paper would like to take this approach, and one of the authors (YK) has proposed a prototype of such a model [1], where there are three Higgs doublets with VEVs of $v_i = \langle H_i^0 \rangle$ satisfying

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2. \tag{1.1}$$

It can lead to the charged-lepton mass relation [2],

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2, \tag{1.2}$$

where $M^e = \text{diag.}(m_e, m_\mu, m_\tau) \propto v_i^2$, which gave a remarkable prediction of $m_\tau = 1776.97$ MeV from the observed values of m_e and m_μ .¹ However, a model with multi-Higgs

¹The observed value is $m_\tau^{\text{obs}} = 1776.99_{-0.26}^{+0.29}$ MeV [3].

doublets causes the large flavor changing neutral current (FCNC) in general [4]. Therefore, we must change the scenario in which the origin of the mass spectrum is separated from the electro-weak symmetry breaking. A typical example is the Froggatt-Nielsen (FN) model [5], which has new scalars ϕ and their VEVs (v_i) induce mass spectrum which has nothing to do with the electro-weak symmetry breaking. For example, in ref. [6], a $U(3)_F$ -flavor symmetry has been introduced, where ϕ is regarded as a $U(3)_F$ -nonet field. Then Yukawa interactions of the charged-lepton sector are induced from a higher dimensional operator $[y_0(\phi_{ik}\phi_{kj}/\Lambda^2)L_jH_dE_i]_F$ in the supersymmetry (SUSY) theory, where Λ is the cutoff scale of this effective Lagrangian.² L and E are $SU(2)_L$ -doublet and singlet lepton superfields, respectively, and H_d is the Higgs doublet which couples to the charged-lepton superfields. This model can avoid the FCNC problem. However, we cannot explain why only the bilinear term of FN field ($\phi_{ik}\phi_{kj}$) is introduced in the superpotential.³

In this paper, we propose another mechanism which naturally induces bilinear form of mass spectrum. For example, the charged-lepton masses are induced through F -components of the superfield ϕ as

$$\left[y_0 \frac{\phi_{ij}^\dagger}{\Lambda^2} L_j H_d E_i \right]_D. \tag{1.3}$$

Since a F -component has mass dimension two, so that eq. (1.3) can be a good candidate to reproduce the mass relation of eq. (1.2).⁴ The vacuum of this model induces the mass relation eq. (1.2) as well as SUSY-breaking, so we can expect fruitful byproducts, such as sfermion mass spectrum.

2. A model

We mainly focus on the lepton sector. An application to the quark sector might be possible. Let us start showing our model.

2.1 Lagrangian

We adopt the following O’Raifeartaigh-type superpotential [9],

$$W(\Phi, A, B) = \sum_{f=u,d} \left(W_\Phi(\Phi_f) + \lambda_A^f \text{Tr}[A_f \Phi_f \Phi_f] + \lambda_B^f \text{Tr}[B_f \Phi_f \Phi_f] - \mu_f^2 \text{Tr}[\xi_f A_f] \right). \tag{2.1}$$

The Kähler potential is set as

$$\begin{aligned} \mathcal{K} &= \mathcal{K}_0 + \mathcal{K}_1 + \mathcal{K}_Y, \tag{2.2} \\ \mathcal{K}_0 &= \sum_{f=u,d} \left(\text{Tr}[A_f^\dagger A_f] + \text{Tr}[B_f^\dagger B_f] + \text{Tr}[\Phi_f^\dagger \Phi_f] + H_f^\dagger H_f \right) + L^\dagger L + E^\dagger E + N^\dagger N, \tag{2.3} \end{aligned}$$

²Reference [7] proposed a similar model but without higher dimensional operators.

³A continuous flavor symmetry induces massless NG bosons, while a discrete flavor symmetry cannot forbid higher order terms, such as ϕ^4 in Z_2 -symmetry.

⁴A similar type of eq. (1.3), $[y'_0 \frac{\phi_{ij}^\dagger}{\Lambda^2} L_j H_u N_i]_D$, was used for tiny Dirac neutrino masses in ref. [8], where N is the right-handed neutrino and H_u is the Higgs doublet which constructs up-type Yukawa interactions.

$$\mathcal{K}_1 = -\frac{1}{\Lambda^2} \sum_{f=u,d} \left(\text{Tr}[A_f^\dagger A_f]^2 + \text{Tr}[B_f^\dagger B_f]^2 \right), \quad (2.4)$$

$$\mathcal{K}_Y = \frac{1}{\Lambda^2} \text{Tr} \left[\left(y_A^d A_d^\dagger + y_B^d B_d^\dagger \right) L H_d E \right] + \frac{1}{\Lambda^2} \text{Tr} \left[\left(y_A^u A_u^\dagger + y_B^u B_u^\dagger \right) L H_u N \right] + \text{h.c.}, \quad (2.5)$$

where Φ_f , A_f and B_f are $U(3)_F$ -nonet superfields. ξ_f is a 3×3 numerical matrix while all other couplings (y 's, λ 's) are complex numbers (not matrices). W_Φ is a superpotential which contains only Φ_f . \mathcal{K}_1 is introduced to realize $\langle A_f \rangle = \langle B_f \rangle = 0$ and also avoids massless scalars as will be shown later.⁵ $\text{Tr}[A_d^\dagger L H_d E]$ and $\text{Tr}[B_d^\dagger L H_d E]$ in \mathcal{K}_Y play crucial roles of generating effective Yukawa interactions for the charged-leptons and neutrinos.

We take R -charge assignments as $R(A_f) = R(B_f) = 2$ and $R(\Phi_f) = 0$ in order to forbid phenomenologically unwanted interactions, $\text{Tr}[A_f^2]$, $\text{Tr}[A_f^3]$, $\text{Tr}[A_f^2 \Phi_f]$, $\text{Tr}[A_d L H_d E]$, $\text{Tr}[B_d L H_d E]$, and so on. We also introduce an additional Z_2 -symmetry, under which only Φ_f transforms as odd-parity fields. It forbids the terms, $\text{Tr}[A_f \Phi_f]$, $\text{Tr}[B_f \Phi_f]$, $\text{Tr}[\Phi_d L H_d E]$, and so on. As for a tadpole term, $\mu'^2 \text{Tr}[\xi_f B_f]$ can be eliminated by the field redefinition of A_f and B_f .⁶ We will omit the indices “ u ” and “ d ” when they are obvious.

2.2 Vacuum of the model

The scalar potential ($V = -\mathcal{L}_{\text{scalar}}$) is given by

$$V = - \left(\text{Tr} \left[F_A \frac{\partial W}{\partial A} \right] + \text{Tr} \left[F_B \frac{\partial W}{\partial B} \right] + \text{Tr} \left[F_\Phi \frac{\partial W}{\partial \Phi} \right] \right). \quad (2.6)$$

F 's are calculated from the equations of motions ($\partial \mathcal{L} / \partial F = 0$) as

$$F_\Phi^\dagger + \frac{\partial W}{\partial \Phi} = 0, \quad (2.7)$$

$$F_A^\dagger + \frac{\partial W}{\partial A} - \frac{2}{\Lambda^2} \left(\text{Tr}[A^\dagger A] F_A^\dagger + \text{Tr}[F_A^\dagger A] A^\dagger \right) + \frac{1}{\Lambda^2} y_A^* \mathcal{X}^\dagger = 0, \quad (2.8)$$

$$F_B^\dagger + \frac{\partial W}{\partial B} - \frac{2}{\Lambda^2} \left(\text{Tr}[B^\dagger B] F_B^\dagger + \text{Tr}[F_B^\dagger B] B^\dagger \right) + \frac{1}{\Lambda^2} y_B^* \mathcal{X}^\dagger = 0, \quad (2.9)$$

where $\mathcal{X} = F_L H E + L F_H E + L H F_E$. Notice that V in eq. (2.6) contains fields L , E and H through the Kähler potential, \mathcal{K}_Y . The equation of motion of F_L^\dagger is given by

$$F_L^\dagger + \frac{\partial W}{\partial L} + \frac{1}{\Lambda} H E (y_A F_A^\dagger + y_B F_B^\dagger) = 0, \quad (2.10)$$

and equations of motions of F_H^\dagger and F_E^\dagger are similar to it.⁷

It is hard to obtain exact solutions of eqs. (2.8) and (2.9), thus we use a perturbation method of expanding $1/\Lambda^n$. Then we obtain

$$F_A^\dagger = -\frac{\partial W}{\partial A} + \frac{2}{\Lambda^2} \left(\text{Tr}[A^\dagger A] \frac{\partial W}{\partial A} + \text{Tr} \left[A \frac{\partial W}{\partial A} \right] A^\dagger \right) + \mathcal{O} \left(\frac{1}{\Lambda^4} \right), \quad (2.11)$$

⁵The minus sign of \mathcal{K}_1 is assumed to be derived by the underlying theory.

⁶Precisely speaking, we should determine the Kähler potential eq. (2.2) after this redefinition.

⁷Of course eqs. (2.8)~(2.10) are also obtained directly from the inverse of the Kähler potential, such as $F_A^\dagger = -\mathcal{K}_{AA^\dagger}^{-1} dW/dA$.

and the similar equation of B (which is calculated by a replacement of $A \rightarrow B$). As eq. (2.1) derives

$$\frac{\partial W}{\partial \Phi} = \frac{\partial W_\Phi}{\partial \Phi} + \lambda_A(A\Phi + \Phi A) + \lambda_B(B\Phi + \Phi B), \quad (2.12)$$

$$\frac{\partial W}{\partial A} = \lambda_A \Phi \Phi - \mu^2 \xi, \quad (2.13)$$

$$\frac{\partial W}{\partial B} = \lambda_B \Phi \Phi, \quad (2.14)$$

the scalar potential becomes

$$V = V_0 + V_1, \quad (2.15)$$

$$V_0 = \text{Tr} \left[\left| \frac{\partial W_\Phi}{\partial \Phi} + \lambda(C\Phi + \Phi C) \right|^2 \right] + \text{Tr} [|\lambda_A \Phi \Phi - \mu^2 \xi|^2 + |\lambda_B \Phi \Phi|^2], \quad (2.16)$$

$$V_1 = \frac{2}{\Lambda^2} \left\{ (|\lambda_A|^2 \text{Tr}[A^\dagger A] + |\lambda_B|^2 \text{Tr}[B^\dagger B]) \text{Tr}[\Phi \Phi \Phi^\dagger \Phi^\dagger] \right. \\ \left. - (\lambda_A^* \mu^2 \text{Tr}[A^\dagger A] \text{Tr}[\xi \Phi^\dagger \Phi^\dagger] + \text{h.c.}) + |\mu^2|^2 \text{Tr}[A^\dagger A] \text{Tr}[\xi^\dagger \xi] \right. \\ \left. + |\lambda_A \text{Tr}[A \Phi \Phi] - \mu^2 \text{Tr}[A \xi]|^2 + |\lambda_B|^2 |\text{Tr}[B \Phi \Phi]|^2 \right\}, \quad (2.17)$$

where C and C' are defined by

$$\lambda C \equiv \lambda_A A + \lambda_B B, \quad \lambda C' = -\lambda_B A + \lambda_A B \quad (2.18)$$

with $\lambda = \sqrt{\lambda_A^2 + \lambda_B^2}$. Notice that V_0 (V_1) is the potential of $\mathcal{O}(\Lambda^0)$ ($\mathcal{O}(\Lambda^{-2})$).

Nine scalars of C' are massless in the tree level potential V_0 , because eq. (2.16) does not contain C' . Thus, in the direction of C' , $\langle A \rangle$ and $\langle B \rangle$ are not determined at $\mathcal{O}(\Lambda^0)$. They are determined by including $\mathcal{O}(\Lambda^{-2})$ corrections of V_1 . Stationary conditions $dV/dA = dV/dB = 0$ decide the vacuum at

$$\langle A \rangle = \langle B \rangle = 0 \quad (2.19)$$

with the condition

$$\frac{\partial W_\Phi}{\partial \Phi} = 0. \quad (2.20)$$

Under the eqs. (2.19) and (2.20), the condition of $dV/d\Phi = 0$ determines $\langle \Phi \rangle$ as

$$\langle \Phi \rangle \langle \Phi \rangle = \frac{\lambda_A}{\lambda_A^2 + \lambda_B^2} \mu^2 \xi. \quad (2.21)$$

Then the height of the scalar potential at this vacuum is given by

$$V_{\min} = \left(1 - \frac{\lambda_A^2}{\lambda_A^2 + \lambda_B^2} \right) \mu^4 \text{Tr}[\xi^\dagger \xi]. \quad (2.22)$$

We will show that the minimum exists at $\langle L \rangle = \langle E \rangle = 0$ in section 5, and then F_Φ^\dagger , F_A^\dagger and F_B^\dagger are given by

$$F_\Phi^\dagger = -\frac{\partial W}{\partial \Phi} = 0, \quad F_A^\dagger = -\frac{\partial W}{\partial A} = \lambda_B \frac{\lambda_B}{\lambda_A} \Phi \Phi, \quad F_B^\dagger = -\frac{\partial W}{\partial B} = -\lambda_B \Phi \Phi, \quad (2.23)$$

from eqs. (2.7)~(2.9), respectively. Thus, the effective Yukawa interaction of the charged-lepton sector is given by

$$(y_A^d \lambda_B^d - y_B^d \lambda_A^d) \frac{\lambda_B^d \langle \Phi_{dik} \rangle \langle \Phi_{dkj} \rangle}{\lambda_A^d \Lambda^2} L_j H_d E_i, \quad (2.24)$$

which is the desirable bilinear form and might produce the mass relation $m_{ei} \propto v_{di}^2$ ($v_{di} = \langle \Phi_{dii} \rangle$) in the diagonal basis of $\langle \Phi_d \rangle$.

Notice that eq. (2.21) seems to imply the flavor indices of $\langle \Phi \rangle^2$ are completely fixed by the parameter ξ . However, it is not correct because eq. (2.20) must be also satisfied at the vacuum. We take a standpoint that the parameter ξ is basically free, and it is constrained by $W_\Phi(\Phi)$ through the relation of eq. (3.6) which will be shown in section 3.

2.3 Mass spectra of $U(3)_F$ -nonet fields

Now let us calculate fermion masses of $U(3)_F$ -nonet particles of Φ , A and B which are denote as ψ_Φ , ψ_A and ψ_B , respectively. The fermion masses are induced from the 2nd and 3rd terms in eq. (2.1) as

$$\mathcal{L}_{\text{mass}} = \lambda \sum_{i,j} (\psi_C)_{ij} (v_i + v_j) (\psi_\Phi)_{ji} + 2\lambda_d \sum_i (\psi_\Phi)_{ii} \sum_j \hat{v}_j (\psi_\Phi)_{jj}, \quad (2.25)$$

where $\hat{v}_i = v_i - (v_1 + v_2 + v_3)/3$ and $v_i = \langle \Phi_{ii} \rangle$. The superfields A and B couple to Φ only through the term $\text{Tr}[(\lambda_A A + \lambda_B B)\Phi\Phi]$ in the superpotential W so that $\psi_{C'}$ are massless while ψ_C have Dirac masses of $\mathcal{O}(\langle \Phi \rangle)$ with ψ_Φ . This situation is not affected by the existence of the Kähler potential \mathcal{K}_1 due to $\langle A \rangle = \langle B \rangle = 0$.

By taking account of R -parity conservation, A and B should be regarded as R -parity even. Thus the massless $\psi_{C'}$ are the lightest superparticle (LSP). We should remind that one degree of freedom in $\psi_{C'}$ is absorbed into the longitudinal mode of the gravitino, and its mass becomes $m_{3/2} \simeq F/M_P$ (M_P : four-dimensional Planck scale). Other eight components of $\psi_{C'}$ are remaining as massless fermions, which seem problematic from the view point of phenomenology. But, is it true? $\psi_{C'}$ interacts with the leptons through the 1st and 2nd terms of eq. (2.5) which contains the interactions

$$\mathcal{L} \ni \frac{1}{\Lambda^2} (\text{Tr}[\partial_\mu \gamma^\mu \psi_{C'_d}^\dagger \psi_L H_d E] + \text{Tr}[\partial_\mu \gamma^\mu \psi_{C'_d}^\dagger L \psi_{H_d} E] + \text{Tr}[\partial_\mu \gamma^\mu \psi_{C'_d}^\dagger L H_d \psi_E]). \quad (2.26)$$

These interactions in the diagrams with $\psi_{C'}$ in the external lines vanish by using the equation of motion, $\partial_\mu \gamma^\mu \psi_{C'_d} = 0$.⁸ Thus, although $\psi_{C'}$ are massless LSP, the decay processes to $\psi_{C'}$ are strongly suppressed. The FCNC processes, such as $\mu \rightarrow e\gamma$, mediated by $\psi_{C'}$ in the loop diagrams are also suppressed due to the small coupling of (lepton)-(slepton)-($\psi_{C'}$) and no chiral-flip of $\psi_{C'}$.

Next let us estimate the scalar masses of nonet fields. The scalar masses of Φ are of order $\langle \Phi \rangle$ which are induced from

$$4\lambda_d^2 \left(\sum_i v_i^2 \right) \left(\sum_j \Phi_{jj} \right)^2 + 2\lambda^2 \sum_{i,j} (v_i v_j + \delta_{ij}) \Phi_{ij} \Phi_{ji}, \quad (2.27)$$

⁸The interactions $\frac{1}{\Lambda^2} (\text{Tr}[\psi_{C'_d}^\dagger \partial_\mu \gamma^\mu \psi_L H_d E] + \text{Tr}[\psi_{C'_d}^\dagger L \partial_\mu \gamma^\mu \psi_H E] + \text{Tr}[\psi_{C'_d}^\dagger L H_d \partial_\mu \gamma^\mu \psi_E])$ also vanish by using partial integral.

in the diagonal basis of $\langle \Phi \rangle$. On the other hand, the masses of C and C' are of orders $\langle \Phi \rangle$ and $\langle \Phi \rangle^2/\Lambda$, respectively, which are induced from

$$2\lambda^2 \left\{ \text{Tr}[\langle \Phi \rangle \langle \Phi \rangle CC + \langle \Phi \rangle C \langle \Phi \rangle C] + \frac{\lambda_B^2}{\lambda_A^2} \frac{\text{Tr}[\langle \Phi \rangle^4]}{\Lambda^2} \left(\text{Tr}[AA^\dagger] + (\text{Tr}[BB^\dagger]) \right) \right\}. \quad (2.28)$$

It should be emphasized that C' scalars become massive through V_1 . Although C and C' interact with sleptons through the Kähler interactions $\frac{1}{\Lambda^2} \text{Tr}[\partial^2 C^\dagger L H_d E]$ and $\frac{1}{\Lambda^2} \text{Tr}[\partial^2 C'^\dagger L H_d E]$, respectively, their contributions to the FCNC processes are negligibly small. It is because these interactions only exit in higher loops as well as C and C' are heavy enough.

3. Lepton mass spectra

The mass relations of quarks and leptons (and also their superpartners) are generated in our model. A main purpose of this paper is to propose a new mechanism of generating the bilinear form of the mass spectrum. So we briefly show the derivation of the charged-lepton mass relation in eq. (1.2).

For our goal, we take the simplest superpotential W_Φ as⁹

$$W_\Phi(\Phi_d) = \lambda_d \text{Tr}[\Phi_d] \left(\text{Tr}[\Phi_d \Phi_d] - \frac{2}{9} (\text{Tr}[\Phi_d])^2 \right) + m_d \text{Tr}[\Phi_d \Phi_d]. \quad (3.1)$$

The form of the cubic term (λ_d -term) in eq. (3.1) is equivalent to

$$\lambda_d \left(\text{Tr}[\Phi_d \Phi_d \Phi_d] - \text{Tr}[\Phi_d^{(8)} \Phi_d^{(8)} \Phi_d^{(8)}] \right), \quad (3.2)$$

where $\Phi_d^{(8)} = \Phi_d - \text{Tr}[\Phi_d]/3$. We assume to drop $\Phi_d^{(8)} \Phi_d^{(8)} \Phi_d^{(8)}$ -term from the $U(3)_F$ -invariant cubic term $\text{Tr}[\Phi_d \Phi_d \Phi_d]$. It might be possible by imposing an additional symmetry as shown in ref. [6].¹⁰

From eq. (3.1), we obtain

$$\frac{\partial W_\Phi}{\partial \Phi_d} = 0 = c_1^d \Phi_d + c_0^d \mathbf{1}, \quad (3.3)$$

where

$$c_1^d = 2(m_d + \lambda_d \text{Tr}[\Phi_d]), \quad (3.4)$$

$$c_0^d = \lambda_d \left(\text{Tr}[\Phi_d \Phi_d] - \frac{2}{3} (\text{Tr}[\Phi_d])^2 \right). \quad (3.5)$$

The coefficients c_1^d and c_0^d in eq. (3.3) must be zero [10] in order to obtain non-zero and non-degenerate eigenvalues of $\langle \Phi_d \rangle$. Then, the condition $c_0^d = 0$ just gives the VEV-relation of eq. (1.1) in the diagonal basis of $\langle \Phi_d \rangle$.

⁹We assume R - and Z_2 -symmetries are (spontaneously) broken only in $W_\Phi(\Phi_f)$ sector, in which $R(\lambda_d) = R(m_d) = 2$ and Z_2 -parity odd λ_d are induced from VEVs of some unknown fields possessing R -charges and Z_2 -parity odd.

¹⁰Another Z_2 -parity with $+1$ for $\Phi_d^{(1)} = \text{Tr}[\Phi_d]/3$ and -1 for $\Phi_d^{(8)}$ worked well.

We should also notice that eqs. (2.20) and (2.21) require one relation among six parameters, $\lambda_A, \lambda_B, \mu_d, \xi_d$, and λ_d, m_d in $W_\Phi(\Phi_d)$ as

$$\frac{\lambda_A \mu_d^2}{\lambda_A^2 + \lambda_B^2} \text{Tr}[\xi_d] = \frac{2}{3} \frac{m_d^2}{\lambda_d^2}. \quad (3.6)$$

Taking the same order of μ_d and m_d might be natural for the setup of the present model.

4. Slepton mass spectra

Now let us investigate the slepton mass spectra in our model. The Kähler potential of $\mathcal{O}(\Lambda^{-2})$ derives the SUSY breaking slepton masses

$$\begin{aligned} \mathcal{K}_L = \frac{1}{\Lambda^2} & \left(\text{Tr} \left[|(y_1^d A_d + y_2^d B_d)L|^2 \right] + \text{Tr} \left[|y_3^d A_d + y_4^d B_d|^2 \right] \text{Tr}[|L|^2] \right. \\ & + \text{Tr} \left[|(y_1^u A_u + y_2^u B_u)L|^2 \right] + \text{Tr} \left[|y_3^u A_u + y_4^u B_u|^2 \right] \text{Tr}[|L|^2] \\ & + \text{Tr} \left[|(y_5^d A_d + y_6^d B_d)E|^2 \right] + \text{Tr} \left[|y_7^d A_d + y_8^d B_d|^2 \right] \text{Tr}[|E|^2] \\ & \left. + \text{Tr} \left[|(y_5^u A_u + y_6^u B_u)N|^2 \right] + \text{Tr} \left[|y_7^u A_u + y_8^u B_u|^2 \right] \text{Tr}[|N|^2] \right) \quad (4.1) \end{aligned}$$

as well as makes $\langle L \rangle$ and $\langle E \rangle$ zero in the scalar potential.

Reminding that the charged-lepton masses M_i^e have been given by

$$M_i^e = (y_A^d \lambda_B^d - y_B^d \lambda_A^d) \frac{\lambda_B^d v_{di}^2}{\lambda_A^d \Lambda^2} \langle H_d \rangle \quad (4.2)$$

from eq. (2.24), the 1st and 2nd terms of eq. (4.1) induce the left-handed slepton masses as

$$\left(\frac{y_1^d \lambda_B^d - y_2^d \lambda_A^d}{y_A^d \lambda_B^d - y_B^d \lambda_A^d} \right)^2 \frac{\Lambda^2}{\langle H_d \rangle^2} \tilde{e}_{Li} (M^e)_i^2 \tilde{e}_{Li} + \left(\frac{y_3^d \lambda_B^d - y_4^d \lambda_A^d}{y_A^d \lambda_B^d - y_B^d \lambda_A^d} \right)^2 \frac{\Lambda^2}{\langle H_d \rangle^2} \tilde{e}_{Li} \tilde{e}_{Li} \sum_k (M^e)_k^2. \quad (4.3)$$

The 1st term induces generation-dependent masses (proportional to the charged-lepton squared masses) while the 2nd term gives the universal soft mass. The neutrino masses M_i^ν are given by the similar equation to eq. (4.2), and the matrix $\langle \Phi_u \rangle$ is not diagonal in the diagonal basis of $\langle \Phi_d \rangle$ because the neutrino mixing matrix U_ν is not $U_\nu = \mathbf{1}$. So the slepton mass matrix from the 3rd and 4th terms in eq. (4.1) is not diagonal in general. Here we take a standpoint that neutrinos are Dirac particles with tiny Dirac masses. It is because an introduction of Majorana masses of right-handed neutrinos heavier than Λ might be unnatural¹¹ (See eq. (4.8)). In this case a small value of μ_u^2 is required which induces the small values of $\langle \Phi_u \rangle^2$ as shown in eq. (2.21), and then non-degenerate effects from the neutrino sector are negligibly small due to the lepton mass relation $m_{\nu i}^2/m_{e_i}^2 < 10^{-14}$ (even

¹¹We might also take another standpoint that the origin of Majorana masses is beyond our model and can be heavier than Λ . In this case the following results are changed.

for the inverted-hierarchical neutrino mass spectrum). Then the contributions from A_u and B_u in the 3rd and 4th terms in eq. (4.1) can be neglected.¹²

Therefore the left-handed slepton masses $(\tilde{m}_{LL})_i^2$ have the form¹³

$$(\tilde{m}_{LL})_i^2 = k_L[(M^e)_i^2 + m_{L0}^2] \quad (4.4)$$

where m_{L0}^2 is the universal soft mass for all three generations. It should be noticed that non-degenerate masses can dominate the universal masses, in which charged-slepton masses are almost proportional to the charged-lepton masses. The right-handed charged-slepton masses are also calculated from eq. (4.1) as

$$(\tilde{m}_{RR})_i^2 = k_R[(M^e)_i^2 + m_{R0}^2]. \quad (4.5)$$

Notice that the left-right mixing masses \tilde{m}_{LR}^2 are zero due to the (approximate) R -symmetry. As for sneutrino sector, right-handed sneutrinos have masses of $\mathcal{O}(M^\nu \Lambda / \langle H_u \rangle)$, for example, right-handed τ -sneutrino has a mass of $\mathcal{O}(100)$ eV in case of $\Lambda \sim 10^5$ GeV. Due to the tiny neutrino Yukawa couplings, the FCNC processes induced by the (s)neutrinos are suppressed. Anyhow the light sneutrinos might be interesting for the cosmology.

We emphasize again that the charged-lepton and charged-slepton mass matrices are diagonalized simultaneously in the flavor space. Therefore, the FCNC processes in the lepton sector, for example $\mu \rightarrow e\gamma$, are suppressed, although the charged-sleptons have non-degenerate masses in general.

As for the gaugino masses, it is difficult to generate the suitable scale of them. It is because the R -symmetry is broken only in W_Φ sector. (Reminding that gaugino masses require both SUSY and R -symmetry breakings, only higher order operators can induce gaugino masses in the present model.) So here we assume that the gaugino masses are induced another source, such as moduli F -terms. The μ -term is also assumed to be induced from another mechanism.

Here let us fix the scale of Λ in our model. We take the soft SUSY breaking masses as

$$F/\Lambda \sim \mathcal{O}(1) \text{ TeV} \quad (4.6)$$

($\tilde{m}_\tau \simeq 1 \text{ TeV}$), while the τ -Yukawa should be

$$F/\Lambda^2 \sim \mathcal{O}(10^{-2}). \quad (4.7)$$

Combining eqs. (4.6) and (4.7), the scale of Λ should be

$$\Lambda \sim \mathcal{O}(10^5) \text{ GeV}. \quad (4.8)$$

One example of derivation Λ is considering the large extra dimensional theory [11], in which the Λ is regarded as the D -dimensional Planck scale M_* with a relation of

$$M_P^2 = M_*^{2+d}(2\pi R)^d, \quad (4.9)$$

¹²We do not consider the case of $(y_i^\nu/y_i^e)^2 > 10^{14}$ ($i = 1, 2, 3$) and accidental cancellations among y 's, λ 's, and $\langle \Phi \rangle$'s.

¹³Here $k_L = [(y_1^d \lambda_B^d - y_2^d \lambda_A^d)/(y_A^d \lambda_B^d - y_B^d \lambda_A^d)]^2 (\Lambda / \langle H_d \rangle)^2$, and so on.

fields	VEV	fermion masses	scalar masses
Φ_f	$\langle\Phi\rangle_f$	$\langle\Phi\rangle_f$	$\langle\Phi_f\rangle$
C_f	0	$\langle\Phi\rangle_f$	$\langle\Phi_f\rangle$
C'_f	0	0	$\langle\Phi_f\rangle^2/\Lambda$

Table 1: Fermion and scalar masses of $U(3)_F$ -nonet fields.

where d is a number of the extra dimension ($D = 4 + d$). $M_* \sim 10^5$ GeV means $R \sim 10^8$ GeV $^{-1}$ ($\sim 10^{-7}$ m) in the case of $D = 6$. (The $D = 5$ case is experimentally excluded.) The present model requires a setup in which all fields except for gravity multiplets are localized on the 4-dimensional brane.

In a short summary we present the mass spectra of the model. The input parameters are $m_\tau \sim 1$ GeV, $m_{\tilde{\tau}} \sim 10^3$ GeV and $m_{\nu 3} \sim 10^{-10}$ GeV. Then the outputs are $\langle\Phi\rangle_d \sim 10^4$ GeV, $\langle\Phi\rangle_u \sim 10^{-1}$ GeV and $\Lambda \sim 10^5$ GeV. We show the order of mass spectra of $U(3)_F$ -nonet fields in table 1.

We should notice that the gravitino is the next-LSP (NLSP), $m_{3/2} \sim 0.1$ eV, and the mass of right-handed τ -sneutrino is of order 100 eV. The lightest right-handed sneutrino is the next-to-next-LSP (NNLSP) in this model.¹⁴

Finally we comment on the case of introducing a soft term

$$V_{\text{soft}} \sim m_{3/2} \mu_f^2 \text{Tr}[\xi_f A_f] + \text{h.c.}, \tag{4.10}$$

in the supergravity (SUGRA) setup. Notice that this term breaks the R -symmetry explicitly, and then the vacuum is shift to non-vanishing $\langle A \rangle$ and $\langle B \rangle \sim \Lambda^2/M_P$. Then massless eight $\psi_{C'}$ gain $\mathcal{O}(m_{3/2})$ masses through the non-canonical Kähler \mathcal{K}_1 in eq. (2.4). In this case the processes which have $\psi_{C'}$ in the external lines are no longer vanished. For example, a slepton decays to a lepton and $\psi_{C'}$, such as $L \rightarrow \psi_{C'_d} + \psi_E$ through the Kähler interaction in eq. (2.26). The partial decay width of this process is roughly estimated¹⁵ as $\Gamma \geq m_{3/2}^2 \langle H_d \rangle^2 \tilde{m}_E / \Lambda^4$. Although the gaugino masses can be generated by the gauge mediation with the additional messenger fields [12], they are too light and still need another SUSY breaking source.

5. Summary and discussions

We have investigated new mechanism which induces flavor mass spectrum by the F -components of $U(3)_F$ -nonet superfields. Fermion masses are generated through the Kähler potential, and then the desired bilinear form of the mass spectrum is realized as $(M^e)_{ij} \propto \sum_k \langle \Phi_{dik} \rangle \langle \Phi_{dkj} \rangle$. This can induce the interesting charged-lepton mass relation eq. (1.2) by using a particular form of $W_\Phi(\Phi_d)$.

In the nonet superfields, eight fermions of $(\psi_{C'})$ remain as massless particles. However smallness of their couplings and no chiral-flip of them in the loop diagrams suppress the

¹⁴The cosmological studies might be interesting.

¹⁵More accurate estimations should be needed to check whether this process disturbs the big bang nucleon synthesis or not.

FCNC processes. The interactions with $\psi_{C'}$ in the external lines vanish so that the decay processes to $\psi_{C'}$ are strongly suppressed.

The charged-slepton mass spectrum is non-degenerate in general and can be even hierarchical (proportional to the charged-lepton masses in the specific case). The FCNCs in the lepton sector are suppressed since the charged-lepton and slepton (except for right-handed sneutrino) mass matrices are diagonalized simultaneously in the flavor space. The right-handed sneutrinos are light with the similar ratio to the lepton sector ($\tilde{m}_{\nu_R}/\tilde{m}_e \sim m_\nu/m_e$). Due to the tiny neutrino effective Yukawa couplings, the FCNC processes induced by the (s)neutrinos are suppressed. The gravitino mass is about 0.1 eV in case of $\Lambda \sim 10^5$ GeV, and the setup of our model fits the large extra dimension scenario. We have also comment on the present model in the SUGRA setup, in which massless $\psi_{C'}$ obtain masses of $\mathcal{O}(m_{3/2})$.

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